

QUANTUM RELATIVITY: AN ESSAY

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ABSTRACT. Is “Gravity” a deformation of “Electromagnetism”?

Deformation theory suggests “quantizing” *Special Relativity*: formulate *Quantum Information Dynamics* (QID) as an $SL_2(C)_h$ -gauge theory of dynamical lattices, with *unifying gauge “group”* the quantum bundle obtained from the *Hopf monopole bundle* underlying the quaternionic algebra and Dirac-Weyl spinors:

$$S^1 \cong SU_1 \xrightarrow{\text{fiber}} S^3 \cong SU_2 \xrightarrow{2:1} SO_3 \leftarrow SO_2$$

$$\quad \quad \quad \swarrow \text{Hopf bundle} \quad \quad \searrow \text{Homogeneous manifold}$$

$$\quad \quad \quad S^2$$

The deformation parameter is the inverse of light speed $\bar{h} = 1/c$, in duality with Planck’s constant h :

$$\text{Hall conductivity: } \frac{h}{e^2} = \alpha \bar{h} \quad : \text{“Cosmological conductivity”}.$$

Then mass m and electric charge q form a complex coupling constant (m, q) , for which the quantum determinant of the quantum group $SL_2(C)_h$ expresses the interaction strength as a linking number 2-form:

$$Qdet \begin{pmatrix} q & im' \\ -im & q' \end{pmatrix} = qq' - e^{-1/\alpha} mm'.$$

There is room for both Coulomb constant k_C and Newton’s gravitational constant G_N , exponentially weaker then the reciprocal of the fine structure constant α :

$$\frac{G_N m_e^2}{k_C e^2} \approx 10^{-54} \quad \leftrightarrow \quad e^{-1/\alpha} \approx 10^{-59}.$$

Thus “Gravity” emerges already “quantum”, in the discrete framework of QID, based on the quantized complex harmonic oscillator: the quantized qubit.

All looks promising, but will the details backup this “grand design scheme”?

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1. INTRODUCTION

From a Computer Science point of view a new paradigm emerges in Mathematical-Physics: *Quantum Information Dynamics* (QID) [14, 30]. It is a physics interface naturally crystallizing from the general principles of the *Digital World Theory* [12], mathematically implemented using graph complexes and their cohomology [19, 6, 13, 8], unifying classical and quantum physics.

QID emphasizes two main points. One is that classical and quantum information are in an “external-internal space” duality (IE-duality), one information emerging from the other: bit from qubit through measurement, and back to qubit through superpositions (e.g. quantum erasure).

The other important aspect is that the role of Electromagnetism (EM) was underestimated, even after the remarkable discovery of Aharonov-Bohm a. a. of the close connection between quantum phase and *classical* EM as an SU_1 -gauge theory.

A closer inspection of *Special Relativity* reveals that not only time is not “absolute” as a conclusion of a Einstein’s critical analysis of the concept of synchronization, but also the concept of “direction” (parallelism) is subject to the same criticism. Therefore an SO_3 -connection is mandatory to make sense of “direction correlation”; the assumption of a metric with its induced Levi-Civita connection is clearly an extra assumption, no longer appropriate in a modern physics dominated by quantum theory.

In conclusion [14], EM as a SU_1 -gauge theory is only a “Hopf fiber” of QID as a SU_2 -Yang-Mills theory, on whatever “space” may be.

This yields a nice *Background Space-Time Independent Theory* (BIT), from which Quantum Mechanics (QM) and Quantum Field Theory (QFT) emerge, through an averaging over possible space-time coordinate systems, which are embeddings in an ambient space-time manifold “a la” *String Theory*. and Heisenberg canonical commutation relations implementing a categorical,

But then, why is there “Gravity” to “spoil” this nice picture?

At the “antipode” of the possibility that gravitation is a global, entropy related phenomenon [27], we will explore the idea that

Gravity is a deformation of Electromagnetism.

Indeed, Lorentz group is a one-parameter “infinitesimal” deformation of Galilei group. In view of General Relativity, it represents the local symmetries of physical processes (classical or quantum). Therefore we believe that one needs to fully deform Galilei group into a quantum group, *not* to “quantize” General Relativity! As hinted above, we claim that “quantization” comes for free, from the discretization process and categorical approach.

2. LINKING NUMBERS AS AN INTERACTION MODEL

In the context of a discrete model, such as QID, particles are not newtonian “points”, but exhibit symmetries which determine how they interact.

For example the electromagnetic interaction can be expressed using linking numbers and topological (cohomological) degree, whether the theory uses manifolds [9, 24, 3, 23, 17], or algebraic chains/cochains and homological algebra methods [2, 13].

There is a long history regarding the debate between describing interactions in physics in terms of particles *separated* from fields (“local physics”), for instance using Maxwell’s equations in the spirit of Newton’s laws ¹:

$$\frac{d}{dt}(mv + qA) = F_{Lorentz} = q[E + v \times B] = -q\nabla(\Phi - qA),$$

or adopting a relational approach (“categorical physics”) as in Neumann’s approach [7], p.400, using a mutual potential P_{ab} and a (geometric) inductance M_{ab} , to express the magnetic force F_{ab} or torque C_{ab} between two closed electric circuits a and b ([25], p.2,5):

$$F_{ab} = -\nabla P_{ab}, \quad P_{ab} = I_a I_b M_{ab}, \quad M_{ab} = \frac{\mu}{4\pi} \int_a \int_b \frac{dl_a \cdot dl_b}{r},$$

$$C_{ab} = -I_a I_b L_{ab}, \quad L_{ab} = \int_a \int_b \frac{dl_a \times dl_b}{r}.$$

The double integrals are directly related to linking number [9], p.114:

$$Link(a, b) = \frac{1}{4\pi} \int_a \int_b \frac{dr' \times (r - r') \cdot dr}{|r - r'|^3},$$

which is obtained by rewriting Biot-Savart law by converting current elements into moving charge elements:

$$Idl = \frac{dq}{dt} dl = dq \frac{dl}{dt} = v dq.$$

Maybe the loops could also be internal S^1 fibers, allowing to represent the electric force too as a linking number, with the appropriate coupling constant $1/\epsilon$.

So, never mind Gravity, what is Electric Force!?

In view of the *duality between mass and electric charge* at the level of the canonical momentum $P = mv + qA$, which is the one quantized and conserved in experiments like Aharonov-Bohm effect, linking EM and QM like Aharonov-Bohm [21], p.384 (see also [20, 18, 26]):

$$Fluxoid : \quad \int_c (mv + qA) ds = h,$$

we should “grade” the interacting particles with their own coupling constants corresponding to their symmetry groups, with hindsight from particle physics: protons e^+ , electrons e^- and hydrogen atoms e^0 , without the hasty assumption that $e^+ = -e^-$.

¹Here A is the vector potential and Φ is the electric potential.

3. A TOY MODEL: THE NEWTON-COULOMB FORCE

Regarding Newton's law modeling the interaction of two "pointwise" bodies:

$$d/dt (mv) = F_{interactions},$$

the left hand side is additive with respect to the particles which constitute the subsystems of the two interacting systems S_1 and S_2 , while the right hand side is multiplicative, corresponding to the "edges" of the bipartite graph representing the two *separate* systems:

$$d/dt \sum_i m_i v_i = -\nabla \sum_{i,j} \frac{q_i q_j}{r}. \quad (1)$$

Here we have assumed that distance has the role of diminishing the interaction strength, conform to a Coulomb potential law. We also assume that there is *no superposition* between interacting pairs, so that a tensorial coupling constant " k_{ij} " is a product $a_i b_j$, which can be absorbed in the definition of the charges q_i .

With only two types of *real* charges $+$ and $-$, let us relax the assumption $e^- = -e^+$ as follows:

$$e^+ = e - \delta/2 > 0, \quad e^- = -e - \delta/2 < 0, \quad e \gg \delta > 0.$$

Indeed, with hindsight from *charge-parity violation*, if we have *chirality*, why not charge-conjugation violation? And since there is a "gravitational attraction", two electrons effectively repel each other slightly stronger than two protons (notice the negative sign in (1)):

$$\text{Electron} - \text{electron} : (e^-)^2 = (-e - \delta/2)^2 = e^2 + e\delta + \delta^2/4,$$

$$\text{Proton} - \text{proton} : (e^+)^2 = (e - \delta/2)^2 = e^2 - e\delta + \delta^2/4,$$

while the electron-proton force is attractive (built in signs!):

$$\text{Electron} - \text{proton} : e^- e^+ = (-e - \delta/2)(e - \delta/2) = -(e^2 - \delta^2/4).$$

This slightly shifts the origin of the real axis by δ , breaking the "left-right" symmetry.

Now the resulting interaction force is consistent with a separation between a "bulk" electric force, and a "residual", possibly gravitational-like force:

$$F = F_E + F_G, \quad F_E \approx \pm e^2.$$

What about "neutral" bodies? The residual interaction between two "neutral" systems, consisting from a pair of opposite charges each, is:

$$HH - \text{atoms interaction} : (e^+)^2 + (e^-)^2 + 2e^+ e^- = \delta^2.$$

Unfortunately it is a repelling force! Then let's treat δ as a result of renormalization, which according to the author is a Hopf algebra deformation [16], and apply the usual Feynman trick of "nudging" δ into the complex plane to avoid "poles":

$$\text{Feynman trick / Wick rotation} : \quad \delta \mapsto i\delta.$$

In fact "charges" are *sources* of singularities, needed for monodromy if translating differential equations in the language of differential Galois theory; so "charges" would translate into complex residues or periods in a theory formulating interactions via linking numbers and topological degrees.

Now assume that for one system with n_+ “positive” charged particles and n_- “negative” charged particles, the total “mass” and “net charges” are defined as:

$$m = n_+ + n_-, \quad q = n_+ - n_-.$$

If we take the real part of everything, with the above complex charge-parity violation:

$$e^+ = e - i\delta/2, \quad e^- = -e - i\delta/2,$$

we recover in the Coulomb law both the electric force and the gravitational force terms:

$$F_{EG} = -[e^2 qq' - \delta^2 mm']/r^2, \quad (2)$$

with δ^2 playing the role of the gravitational constant G_N .

4. DEFORMING SPECIAL RELATIVITY: $SL_2(C)_h$

² One direct way to break the symmetry between positive and negative charges, is to deform the symmetry group, so that by Noether Theorem in the Lagrangian formalism, to get an asymmetry between the corresponding charges and conserved currents associated with the symmetries.

Quantum computing corresponds to Special Relativity via Klein correspondence of Twistor Theory [11] (Hermitian model): “ $2 + 2^* = 1 + 3$ ”. The corresponding group controlling QC, SR and other models of Quantum Gravity is $SL_2(C)$, with Lie algebra $sl_2(C) = su_2(R) + su_2$.

On the “ $2 + 2^*$ ” side, the quaternions are a generalized complex structure [10]:

$$H = (T^*C, \omega) = C \oplus C^*, \quad J : H \rightarrow H, J^2 = -1,$$

“hosting” qubits / SU_2 and the Hopf bundle (complex harmonic oscillator), while the “ $1 + 3$ ” side can be viewed at the infinitesimal level as a central extension of the angular velocity Lie algebra [1], with the cross-product as a Lie bracket:

$$R \rightarrow H \rightarrow g = (R^3, \times).$$

We interpret it as an infinitesimal deformation, with deformation parameter $\bar{h} = 1/c$:

$$H \ni q = t + v\bar{h}, \quad v \in R^3.$$

Now quantize the symplectic group $H = T^*C$, as the complex version of the Heisenberg group viewed as a central extension:

$$R \rightarrow Heisenberg \rightarrow (T^*R, \omega),$$

by using the Backer-Campbell-Hausdorff formula (with $[\cdot, \cdot]$ in place of \times):

$$q \oplus q' = q + q' + \frac{1}{2}[q, q']\bar{h} + \frac{1}{12}([q, [q, q']] - [q', [q', q]])\bar{h}^2 + \dots$$

On the “ $2 + 2^*$ ” side it should correspond to quantizing $SL_2(C)$ as a quantum group.

Surprisingly enough, if we factor Coulomb’s constant in (2), the coefficient in the Coulomb-Newton force looks like the corresponding *quantum determinant* [28]:

$$qq' - \delta^2/e^2 mm' \quad \leftrightarrow \quad Qdet \begin{pmatrix} q & im' \\ -im & q' \end{pmatrix} = qq' - e^{-h} mm'.$$

²Due to limitations of the essay we will be brief here.

The “flip” in the second column could be due to a complex rotation in a Kahler form.

Moreover the deformation factor e^{-h} could play the role of the universal gravitational constant, explaining why gravity is so weak compared to EM! (see Abstract).

In [15] we suggest the possibility that the two deformation parameters, the conductivity of interactions $\bar{h} = 1/c$ and Planck’s constant h , are in duality:

$$\text{Hall conductivity : } \frac{h}{e^2} = \alpha \bar{h} \quad : \text{Cosmological “conductivity”}.$$

This would just express the “amazing duality” between *Kepler’s Problem* and *Harmonic Oscillator* [1], which we should take as a “sign” that micro-cosmos and macro-cosmos are indeed dual. But is this a “T-duality” $\bar{h} = 1/c$ or “S-duality” $\bar{h} = 1/\alpha$, in the spirit of String Theory [31])?

5. CONCLUSIONS

The weakness of gravity is taken as a hint that gravity is a deformation of “Electromagnetism”, corresponding to a charge-conjugation violation due to non-commutativity and chirality of the unifying gauge “group”, the Hopf monopole bundle. Quantizing it as a complex harmonic oscillator, allows to represent the Newton-Coulomb force as an $SL_2(C)_h$ quantum determinant!

This is a research avenue, the author believes, it is worth pursuing.

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